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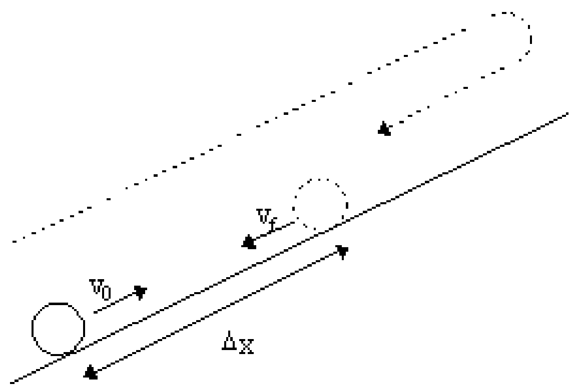
Physics 1120: 1D Kinematics Solutions

1. Initially, a ball has a speed of 5.0 m/s as it rolls up an incline. Some time later, at a distance of 5.5 m up the incline, the ball has a speed of 1.5 m/s DOWN the incline.
   (a) What is the acceleration? What is the average velocity? How much time did this take?
   (b) At some point the velocity of the ball had to have been zero. Where and when did this occur?

   A well-labeled sketch usually helps make the problem clearer.



   (a) Next, we list the list the given information and what we are looking for:

   $v_0 = +5.0$ m/s

   $v_f = -1.5$ m/s

   $\Delta x = +5.5$ m

   a = ?

   $v_{average} = ?$

   t = ?

   Note that I have taken the direction up the incline as positive and that the signs are explicitly stated. It is a very common source of error to leave out or to not consider the signs of directions of all vector quantities.

   To find the acceleration, we find the kinematics equation that contains a and the given quantities. Examining our equations we see that we can use $2a\Delta x = v_f^2 - v_0^2$. Rearranging this equation to find a

   yields $a = \dfrac{v_f^2 - v_0^2}{2\Delta x} = \dfrac{(-1.5 m/s)^2 - (5.0 m/s)^2}{2 \times 5.5 m} = -2.068 m/s^2$. Notice that the acceleration is negative. This means that the acceleration points down the incline. It means that an object traveling up an incline will slow, turn around, and roll down the incline.

The average velocity is defined $v_{average} = \frac{v_0 + v_f}{2} = 1.75 m/s$.

To find the time, we find the kinematics equation that contains a and the given quantities. Examining our equations we see that we can use $\Delta x = \frac{v_0 + v_f}{2} t$. Rearranging this equation to find t yields

$$t = \frac{2\Delta x}{v_0 + v_f} = 3.143 s.$$

(b) When an object moving in 1D turns around we know that the object is instantaneously at rest and that its velocity at that point is $v_3 = 0$. The information that we know is thus:

$v_0 = +5.0$ m/s	
$v_3 = 0$ m/s	This is our new final velocity
$a = -2.068$ m/s^2	From part (a)
$\Delta x = ?$	
$v_{average} = ?$	
$t = ?$	

Notice that the acceleration is a constant of the motion; it has the same value in both parts of the problem.

To find the displacement from the initial position where the ball turns around, we find the kinematics equation that contains x and the given quantities. Examining our equations we see that we can use $2a\Delta x = v_f^2 - v_0^2$. Rearranging this equation to find x yields

$\Delta x = \frac{v_f^2 - v_0^2}{2a} = \frac{(0 m/s)^2 - (5.0 m/s)^2}{2 \times (-2.068 m/s^2)} = 6.04 m$. Notice that this value is bigger than the original 5.5 m and is consistent with the sketch, i.e. the ball was farther up the incline when it turned around.

To find the time it takes for the ball to reach the point where it turns around, we find the kinematics equation that contains t and the given quantities. Examining our equations we see that we can use $v_f = v_0 + at$. Rearranging this equation to find t yields $t = \frac{v_f - v_0}{a} = \frac{(0 m/s) - (5.0 m/s)}{-2.068 m/s^2} = 2.42 s$

.Notice that this value is smaller than the time in part (a) and is consistent with the sketch, i.e. the ball hasn't come back down the incline yet.

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2. A bullet in a rifle accelerates uniformly from rest at a = 70000 m/s^2. If the velocity of the bullet as it leaves the muzzle is 500 m/s, how long is the rifle barrel? How long did it take for the bullet to travel the length of the barrel? What is the average speed of the bullet?

To solve this problem, we list the list the given information and what we are looking for:

$v_0 = 0.0$ m/s	since the bullet is initially at rest
$v_f = 500$ m/s	velocity of the bullet as it leaves the barrel
a = 70,000 m/s	
$\Delta x = ?$	the length of the barrel
t = ?	the time it takes to travel the barrel
$v_{average} = ?$	

To find the length of the barrel, we find the kinematics equation that contains x and the given quantities. Examining our equations we see that we can use $2a\Delta x = v_f^2 - v_0^2$. Rearranging this equation to find a

yields $\Delta x = \dfrac{v_f^2 - v_0^2}{2a} = \dfrac{(500m/s)^2 - (0.0m/s)^2}{2 \times 70000m/s^2} = 1.79m$.

To find the time it takes for the bullet to travel the length of barrel, we find the kinematics equation that contains t and the given quantities. Examining our equations we see that we can use $v_f = v_0 + at$.

Rearranging this equation to find t yields $t = \dfrac{v_f - v_0}{a} = \dfrac{(500m/s) - (0.0m/s)}{70000m/s^2} = 7.1 \times 10^{-3}s$.

The average velocity is defined $v_{average} = \dfrac{v_0 + v_f}{2} = \dfrac{0m/s + 500m/s}{2} = 250m/s$.

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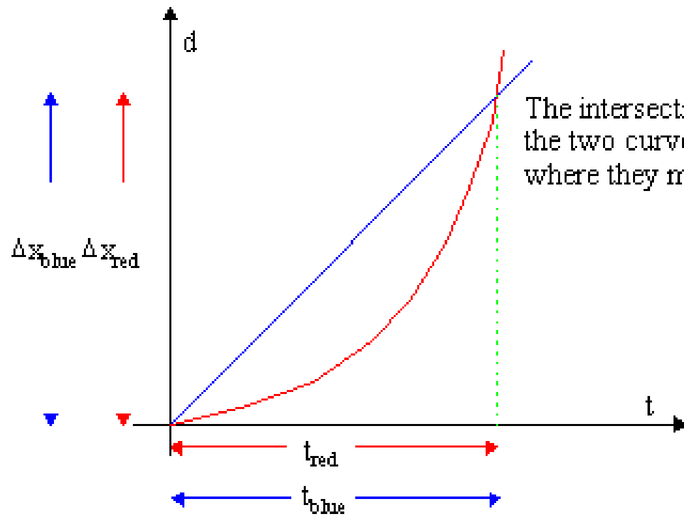
3. A red car is stopped at a red light. As the light turns green, it accelerates forward at 2.00 m/s^2. At the exact same instant, a blue car passes by traveling at 62.0 km/h. When and how far down the road will the cars again meet? Sketch the d versus t motion for each car on the same graph. What was the average velocity of the red car for this time interval? For the blue car? Compare the two and explain the result?

To solve this problem, we list the list the given information

Red Car	**Blue Car**
$v_{0\,red} = 0.0$ m/s	$v_{0\,blue} = 62.0$ km/h = 17.222 m/s
$a_{red} = 2.00$ m/s^2	$a_{blue} = 0$ m/s^2 (constant velocity)
$\Delta x_{red} = ?$	$\Delta x_{blue} = ?$
$t_{red} = ?$	$t_{blue} = ?$

This is an example of a two-body constrained kinematics problem. Even if a sketch was not explicitly required, we would need one anyway to get the constraints. For the sketch, recall that on a d versus t curve an object moving forward with a uniform acceleration should be represented by a line curving upwards while an object with constant forward velocity is represented by a straight line with a positive slope.



Looking at the sketch, we see that our constraints are:

$$\Delta x_{red} = \Delta x_{blue} \text{ (1), and}$$

$$t_{red} = t_{blue} \text{ (2). .}$$

To solve the problem, we must find the kinematics equation that contains the known quantities, v_0 and a, and the unknown quantities, Δx and t. Examining our equations we see that we can use $\Delta x = v_0 t + \frac{1}{2}at^2$. We substitute this equation into both sides of equation (1). This yields,

$$v_{0\ red}t_{red} + \tfrac{1}{2}a_{red}(t_{red})^2 = v_{0\ blue}t_{blue} + \tfrac{1}{2}a_{blue}(t_{blue})^2.$$

We then use equation (2) to replace t_{red} and t_{blue} by t,

$$v_{0\ red}t + \tfrac{1}{2}a_{red}t^2 = v_{0\ blue}t + \tfrac{1}{2}a_{blue}t^2.$$

Plugging in the values of the given quantities yields,

$$\tfrac{1}{2}(2.00)t^2 = 17.2t.$$

The solution of this equation is t = 17.222 seconds. This is the time that elapses before the two cars meet again.

With a value for t, we can find how far down the road the red car has traveled;

$$\Delta x_{red} = v_{0\ red}t + \tfrac{1}{2}a_{red}t^2 = \tfrac{1}{2}(2.00)(17.2)^2 = 297 \text{ m.}$$

As a check, we can find how far down the road the blue car has traveled;

$$\Delta x_{blue} = v_{0\ blue}t + \tfrac{1}{2}a_{blue}t^2 = (17.2)(17.2) = 297 \text{ m.}$$

So the cars meet 297 m down the road.

According to our definition of average velocity, $v_{average\ red} = \Delta x_{red}/t = (297\ m)/(17.2\ s) = 17.2\ m/s$. Since the blue car maintains a constant velocity, $v_{average\ blue} = v_{0\ blue} = 17.2\ m/s$. The two quantities are the same since the two cars have traveled the same distance in the same amount of time.

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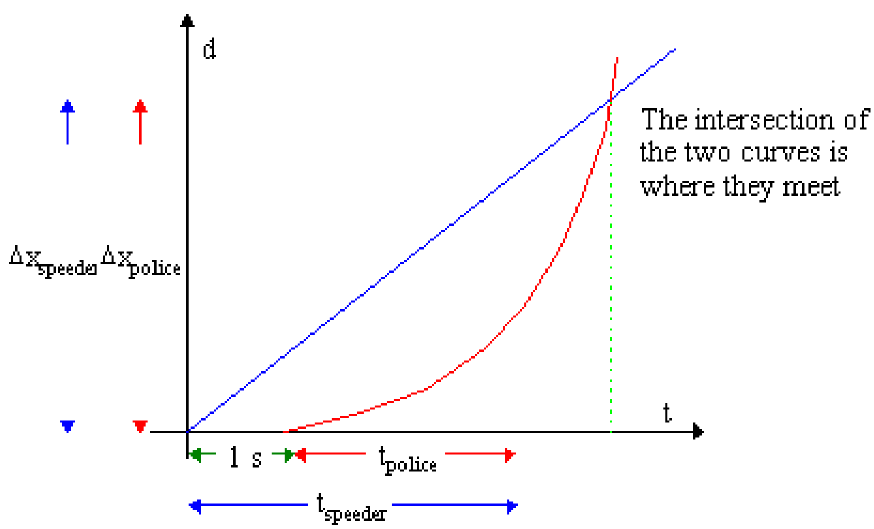
4. A speeding motorist traveling down a straight highway at 110 km/h passes a parked patrol car. It takes the police constable 1.0 s to take a radar reading and to start up his car. The police vehicle accelerates from rest at 2.1 m/s^2. When the constable catches up with the speeder, how far down the road are they and how much time has elapsed since the two cars passed one another?

To solve this problem, we list the list the given information

Constable	**Motorist**
$v_{0\ police} = 0.0\ m/s$	$v_{0\ speeder} = 110\ km/h = 30.556\ m/s$
$a_{police} = 2.00\ m/s^2$	$a_{speeder} = 0\ m/s^2$ (constant velocity)
$\Delta x_{police} = ?$	$\Delta x_{speeder} = ?$
$t_{police} = ?$	$t_{speeder} = ?$

This is an example of a two-body constrained kinematics problem. We need a sketch to get the constraints. For the sketch, recall that on a d versus t curve an object moving forward with a uniform acceleration should be represented by a line curving upwards while an object with constant forward velocity is represented by a straight line with a positive slope.



Looking at the sketch, we see that our constraints are:

$$\Delta x_{speeder} = \Delta x_{police}\ (1), \text{ and}$$

$$t_{speeder} = t_{police} + 1 \text{ (2)}. .$$

To solve the problem, we must find the kinematics equation that contains the known quantities, v_0 and a, and the unknown quantities, Δx and t. Examining our equations we see that we can use $\Delta x = v_0 t + ½at^2$. We substitute this equation into both sides of equation (1). This yields,

$$v_{0\,speeder} t_{speeder} + ½a_{speeder}(t_{speeder})^2 = v_{0\,police} t_{police} + ½a_{police}\,(t_{police})^2.$$

We then use equation (2) to replace $t_{speeder}$ by $t_{police} + 1$,

$$v_{0\,speeder}\,(t_{police} + 1) + ½a_{speeder}(t_{police} + 1)^2 = v_{0\,police} t_{police} + ½a_{police}\,(t_{police})^2.$$

Plugging in the values of the given quantities yields,

$$(30.556)(\,t_{police} + 1) = ½(2.1)(t_{police})^2.$$

This is a quadratic in t_{police}. Solving the quadratic yields, $t_{police} = 30.07$ seconds. It takes the police constable 30.1s to catch up with the speeder. The speeder was traveling for 31.1 s.

With a value for t_{police}, we can find how far down the road the police car has traveled;

$$\Delta x_{police} = v_{0\,police} t_{police} + ½a_{police}\,(t_{police})^2 == ½(2.1)(30.07)^2 = 949 \text{ m.}$$

As a check, we can find how far down the road the speeder's car has traveled;

$$\Delta x_{speeder} = v_{0\,speeder}\,(t_{police} + 1) + ½a_{speeder}(t_{police} + 1)^2 = 30.55631.07 = 949 \text{ m.}$$

So the cars meet 949 m down the road.

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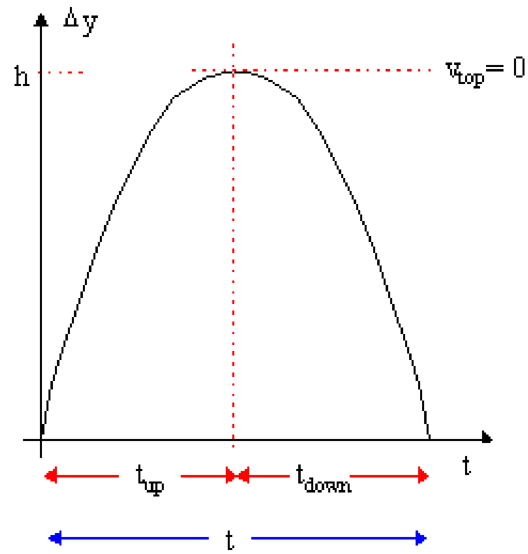
5. A ball is thrown up into the air with an initial velocity of 12.0 m/s. How long will it be in air before it returns to its starting height? To what maximum height will it rise?

To solve this problem, we list the list the given information and what we are looking for:

$v_0 = 12.0$ m/s	velocity as it leaves the hand
$v_{top} = 0$ m/s	since it turns around
$v_f = -12.0$ m/s	symmetry says it must have this value when it returns to the same height
$a = -9.81$ m/s	only gravity is acting
$\Delta y = 0$	since it returns to the same height
$t_{air} = ?$	the time it takes for the entire trip

$t_{up} = t_{down} = ½t_{air} = ?$ symmetry requires this



We have lots and lots of information from symmetry. To find t_{air}, choose the kinematics equation that has t and the known quantities v_0, v_f, and a, that is $v_f = v_0 + at_{air}$. Solving yields $t_{air} = (v_f - v_0)/a = (-v_0 - v_0)/(-g) = 2v_0/g = 2.4465$ seconds. Hence $t_{up} = t_{down} = 1.2232$ s.

To find h, choose the kinematics equation that has Δy (h is a displacement) and the known quantities v_0, v_{top}, and a, that is $2a\Delta y = v_{top}^2 - v_0^2$. Upon rearrangement, this yields $h = \Delta y = (v_0)^2/g = 7.34$ m.

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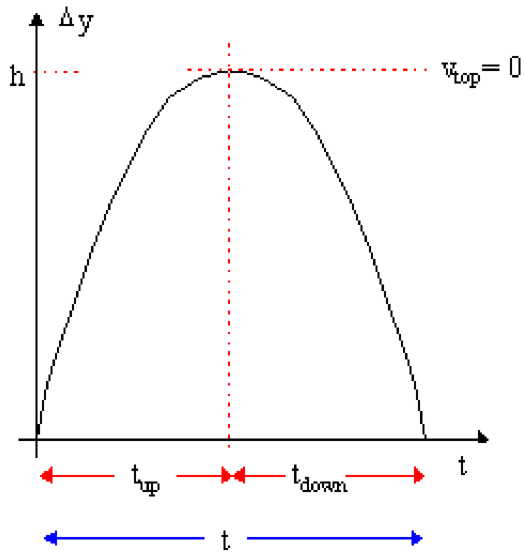
6. A ball is thrown up into the air and returns to the same level. It is in the air for 3.20 seconds. With what initial velocity was it thrown? How high did it rise?

To solve this problem, we list the list the given information and what we are looking for:

$v_0 = ?$	velocity as it leaves the hand
$v_{top} = 0$ m/s	since it turns around
$v_f = -v_0$	symmetry says it must have this value when it returns to the same height
$a = -9.81$ m/s	only gravity is acting
$\Delta y = 0$	since it returns to the same height
$t_{air} = 3.20$ s	the time it takes for the entire trip
$t_{up} = t_{down} = ½t_{air} = 1.60$ s	symmetry requires this



We have lots and lots of information from symmetry. To find v_0, choose the kinematics equation that has v_0 and the known quantities, $v_f = -v_0$, t_{air} and a, that is $v_f = v_0 + at_{air}$. Eliminating v_f yields $-v_0 = v_0 - gt_{air}$. Rearranging gives $v_0 = gt_{air}/2$ = 15.7 m/s.

To find h, choose the kinematics equation that has Δy (h is a displacement) and the known quantities v_0, v_{top}, and a, that is $2a\Delta y = v_{top}^2 - v_0^2$. Upon rearrangement, this yields $h = \Delta y = (v_0)^2/g$ = 12.6 m.

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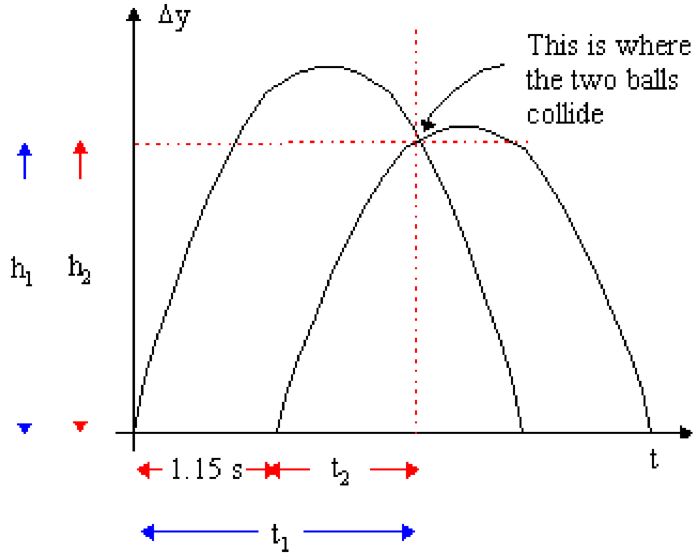
7. Two balls are thrown upwards from the same spot 1.15 seconds apart. The first ball had an initial velocity of 15.0 m/s and the second was 12.0 m/s. At what height do they collide?

To solve this problem, we list the list the given information

Ball #1	**Ball #2**
$v_{0\,1}$ = 15.0 m/s	$v_{0\,2}$ = 12.0 m/s
a_1 = -9.81 m/s^2	a_2 = -9.81 m/s^2
Δy_1 = ?	Δy_2 = ?
t_1 = ?	t_2 = ?

This is an example of a two-body constrained kinematics problem. We need a sketch to get the constraints. For the sketch, recall the shape of the d versus t curve for an object thrown up into the air - a parabola.



Looking at the sketch, we see that our constraints are:

$$\Delta y_1 = \Delta y_2 \text{ (1), and}$$

$$t_1 = t_2 + 1.15 \text{ (2). .}$$

To solve the problem, we must find the kinematics equation that contains the known quantities, v_0 and $a = -g$, and the unknown quantities, Δy and t. Examining our equations we see that we can use $\Delta y = v_0 t - \frac{1}{2}gt^2$. We substitute this equation into both sides of equation (1). This yields,

$$v_{01}t_1 - \tfrac{1}{2}g(t_1)^2 = v_{02}t_2 - \tfrac{1}{2}g(t_2)^2.$$

We then use equation (2) to replace t_1 by $t_2 + 1.15$,

$$v_{01}(t_2+1.15) - \tfrac{1}{2}g(t_2+1.15)^2 = v_{02}t_2 - \tfrac{1}{2}g(t_2)^2.$$

This reduces to

$$1.15v_{01} + v_{01}t_2 - \tfrac{1}{2}g[(t_2)^2 + 2.30t_2 + 1.3225] = v_{02}t_2 - \tfrac{1}{2}g(t_2)^2.$$

Upon rearrangement this becomes

$$(v_{01} - v_{02} - 1.15g)t_2 = -(1.15v_{01} - 0.66125g) .$$

Thus $t_2 = 1.2997$ s, and $t_1 = 2.4497$ s. Now that we have the time that each ball is in the air, we can now find h

$$h = v_{01}t_1 - \tfrac{1}{2}g(t_1)^2 = (15.0)(2.4497) - \tfrac{1}{2}g(2.4497)^2 = 7.31 \text{ m},$$

and double-checking our result